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රුෂීම ස්රඩුප් **GANITHA CHANDRIKA**

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పాఠకులందరికీ నమస్సుమాంజలి. ఈసారి రెందు సంచికలు 3,4 (2022) (July-Dec) కలిపి (పచరించ బడుచున్నది. గమనించ (ప్రార్ధన. అనేక కారణాల వలన online (పచురణ చేస్తున్నాము. పాఠకులంతా అర్థం చేసుకుంటారని అశిస్తున్నాము. మీ అమూల్య అభిప్రాయాలను 2 పేజీలకు మించని గణితాంశాలను ఆహ్వానిస్తున్నాము. విషయాలన్నీ స్కూలు విద్యార్థులకు అనువుగా ఉండాలని గ్రహించగలరు.ఈ సంపుటిలో యథా[పకారం చక్కటిగణిత విషయాలను పొందుపరచినాము. పాఠకులంతా ఆస్వాదిస్తారని భావిస్తున్నాము.

నమస్సులతో

Dr. B.B. రామశర్మ ప్రధాన సంపాదకులు

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ERATOSHTHENES Greek Mathematician

Erathosthenes of Cyrene was a Greek polymath : a mathematician, geographer, poet, astronomer, and music theorist. He was a man of learning, becoming the chief librarian at the Library of Alexandria.

				Ke	ey to	MS	ET =	202	2 Q	uest	ions				
Class						(Que	stio	ns						
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
V	3	3	4	1	1	1	1	1	4	1	4	1	3	4	2
VI	1	2	2	3	2	1	4	4	3	3	4	3	3	1	2
VII	4	1	4	1	2	2	2	4	2	2	4	3	1	3	4
VIII	4	4	2	2	4	4	1	1	1	4	1	2	4	2	2
X	2	1	2	1	3	3	1	3	2	1	2	1	4	1	2
Х	1	2	3	1	4	2	2	1	2	2	3	1	3	2	3

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Brain Teasers for OLYMPIADS

- DR. B.B. RAMA SARMA

1. Solve the system of equation

$$x + \log \left(x + \sqrt{x^2 + 1} \right) = y;$$

$$y + \log \left(y + \sqrt{y^2 + 1} \right) = z$$

$$z + \log \left(z + \sqrt{z^2 + 1} \right) = x.$$

2. Solve the system

log[2xy] = log x log y,log [yz] = log y log z, log [2zx] = log z log x.

3. Find all the solutions to the system of equation



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4. Find ax⁵ + by⁵, if the numbers a,b,x and y satisfy the system of equations

ax + by = 3, $ax^2 + by^2 = 7$, $ax^3 + by^3 = 16$, $ax^4 + by^4 = 42$

5. Solve the system

$$x + \frac{2}{x} = 2y$$
, $y + \frac{2}{y} = 2z$, $z + \frac{2}{z} = 2x$

6. Solve the system of equations

 $(x+y)^3 = z$, $(y+z)^3 = x$, $(z+x)^3 = y$

7. Solve the system

$$x^{2} - |x| = |yz|$$
, $y^{2} - |y| = |zx|$, $z^{2} - |z| = |xy|$

8. Solve the system of equations

 $x + [y] + \{z\}=1.1$, $[x] + \{y\} + z = 2.2$, $\{x\}+y+[z] = 3.3$ where [] and {} denote respectively the greatest integer fuction and the fractional part function respectively.

9. Find the real numbers a for which there exist non-negative real numbers x_1 , x_2 , x_3 , x_4 , x_5 satisfying the system

$$\sum_{k=1}^{5} kx_{k} = a , \qquad \sum_{k=1}^{5} k^{3}x_{k} = a^{2} , \qquad \sum_{k=1}^{5} k^{5}x_{k} = a^{3}$$

10. Solve the system

ax + by

$$= (x-y)^2$$
, by $+ cz = (y-z)^2$,

 $cz+ax = (z-x)^2$ where a,b,c > 0

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Explanations

It is not difficult to guess that x = y = z = 0 is a solution.
 Let us see whether there are other solutions. If x > 0,

then log $\left(x + \sqrt{x^2 + 1}\right) > 0$ and from the first equation

we deduce y > x > 0. From the second and the third equations we obtain x > z > y > x > 0, which is impossible.

If x < 0, then
$$x + \sqrt{x^2 + 1} = \frac{1}{-x + \sqrt{x^2 + 1}} < 1$$

Hence, y < x < 0 and consequently x < z < y < x < 0. which is again impossible. Therefore, the only solution is x = y = z = 0.

2. We have log (2xy) = log 2 + log x + log y. By moving the logarithms containing variables to the right and adding 1 to each side of the three equations, we obtain

 $\log 20 = (\log x - 1) (\log y - 1),$

$$\log 20 = (\log z - 1) (\log x - 1)$$

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Multiplying all equations and taking the square root yields $\pm \log 20 = (\log x - 1) (\log y - 1) (\log z - 1)$ This combined with the equality $\log 20 = (\log x - 1) (\log y - 1)$ Shows that $\log z - 1 = \pm 1$. the other equations gives $\log x - 1 = \pm \log 20$ and $\log y - 1 = \pm 1$ and we obtain the two solutions to the system (200, 100, 100) and $\left(\frac{1}{2}, 1, 1\right)$ 3. The solution very similar to the one we gave for problem 1. We start by observing that the function $f: (0, \infty) \rightarrow \infty$ $(0, \infty)$, f(t) = 4t² / (4t² + 1) is strictly increasing. Hence, if x < y, then f(x) < f(y) so y < z. Repeating the argument we obtain z < x; hence x < y < z < x, which is impossible, Similarly, x > y leads to a contradiction. Therefore, x = y = z. Solving the equation $\frac{4t^2}{4t^2+1} = t$ yields t = 0 or t = $\frac{1}{2}$. Hence, the only triples that satisfy the system are (0,0,0) and $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$

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4. For n = 2 and n = 3, the identity

$$(ax^{n}+by^{n})(x+y)-(ax^{n-1}+by^{n-1})xy=ax^{n+1}+by^{n+1}$$

leads to the equations

7(x+y) - 3xy = 16 and 16(x+y)-7xy = 42Solving these two equations simultaneously yields x + y = -14 and xy = -38Applying the recurrence identity for n = 4 gives $ax^5 + by^5 = (42) (-14) - (16) (-38) = -588 + 608 = 20$ 5. Let (x,y,z) be a solution, Clearly, if one of these numbers is positive, the other two must be positive as well. Multiplying by -1 if necessary, we may assume that x,y,z > 0. Adding the three equations, we obtain

$$x + y + z = 2\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)$$

Also, applying the AM.GM inequality to each equation of the system yields $2x \ge 2\sqrt{2}, 2y \ge 2\sqrt{2}, 2z \ge 2\sqrt{2}$. The shows that in the above equation the left side is greater

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than or equal to $3\sqrt{2}$, while the right side is less than or equal $3\sqrt{2}$. To obtain equality, we must have $x = y = z = \sqrt{2}$, which give one solution. The other solution is obtained by changing sign and is $x = y = z = -\sqrt{2}$. Remark : This is a system of the form

y = f(x), z = (f(y), x = f(z), where f(t) =
$$\frac{1}{2} \left(t + \frac{2}{t} \right)$$

The sequence given by $t_0 \in R$, $t_{n+1} = f(t_n)$, $n \ge 0$ is traditionally used to compute $\sqrt{2}$ with great precision because it converges really to it. No matter what $t_0 \in R$ is, each subsequent term is greater than or equal to $\sqrt{2}$ in absolute value. If for definiteness, $t_0 > 0$, then t_n $\ge \sqrt{2}$ for $n \ge 1$ and also $t_1 \ge t_2 \ge ...$ A term in this sequence can repeat only if it is exactly $\sqrt{2}$. There is no difficulty in solving the analogous system with any number of variables.

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6. Subtracting the second equation from the first, we obtain (x-z) [(x+y)² + (x+y)(y+z) + (y+z)²] = z - x Since, (x+y)² + (x+y)(y+z)+(y+z)² > 0. We obtain x = z. By symmetry, y = z and we are left with solving the equation 8x³ = x. This equation has the solutions x = 0 and x =

 $\pm \frac{1}{2}$. It follows that the solutions to the given system of

equations are x = y = z = 0.

$$x = y = z = \frac{1}{(2\sqrt{2})}$$
 and $x = y = z = -\frac{1}{(2\sqrt{2})}$

7. Let (x,y,z) be solution. If xyz ≠ 0, then since the absolute value is positive, we obtain x² > |yz|, y² > |zx| and z² > |xy|, which by multiplication gives x²y²z² > x²y²z², a contraction. Thus, one of the numbers is zero and using the equation that contains it on the left side we obtain that another of the three number must be zero as well. The third one can be only 0 or ±1. Thus, the solutions are (0,0,0), (1,0,0), (0,0,1), (-1, 0, 0), (0, -1,0) and (0,0,-1).

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We have
$$\sum_{k^2 \le 0} (a - k^2) kx_k = \sum_{k^2 > 0} (a - k^2) kx_k$$

$$\sum_{k^{2} \leq 0} \left(a - k^{2}\right) k^{3} x_{k} = \sum_{k^{2} > 0} \left(a - k^{2}\right) k^{3} x_{k}$$

But
$$\sum_{k^2 \le 0} (a-k^2)k^3 x_k \le a \sum_{k^2 \le 0} (a-k^2)k x_k$$

(11)

$$a\sum_{k^2>0} (a-k^2)kx_k \le \sum_{k^2>0} (a-k^2)k^3x_k$$

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=

Since, the first and the last terms are equal, all inequality signs are in fact equalities, We have

$$\sum_{k^{2}>0} a(a-k^{2})kx_{k} = \sum_{k^{2}>0} k^{2}(a-k^{2})kx_{k}$$

But for $K^2 > a$, we have $a(k^2 - a) kx_k > k^2(k^2 - a) kx_k$ which combined with the inequality above shows that for $k^2 > a x_k = 0$. A similar argument shows that $x_k = 0$ if $k^2 < a$ So, for the system to admit a non-trivial solution, a must be equal to one of the perfect squares 1, 4, 9, 16,25 Note that if $a = m^2$ for some m = 1,2,3,4 or 5, then $x_i = 0$ for $k \neq m$ and $x_m = m$ is a solution.

Alternatively As before, let x_1 , x_2 , x_3 , x_4 , x_5 be a nontrivial solution. From the equations of the system it follows that



 $\left(\sum_{k=1}^{5} k^3 \mathbf{x}_k\right) \leq \left(\sum_{k=1}^{5} k \mathbf{x}_i\right) \left(\sum_{k=3}^{5} k^5 \mathbf{x}_k\right)$

On the other hand, the Cauchy-schwazz inequality applied to the sequences $\left(\sqrt{kx_k}\right)_{k=2}$2 and $\left(\sqrt{k^5x_k}\right)_{k=1}$5 gives

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The relation we deduced above shows that we have equality in the Cauchy-Schwarz inequality and hence the two sequences are proportional. For $x_i = 0$ we have $\sqrt{k^5 x_i} / \sqrt{k x_i} = k^2$ and since all these values are distinct it follows that $x_k = 0$ for exactly one k. As before we conclude that the only possible values for a are 1, 4, 9, 16, 25.

10. Add the third equation to the first and subtract the second to obtain

 $2ax = (x-y)^{2} + (z-x)^{2} - (y-z)^{2} = 2(x^{2} - xy - xz + yz)$ Factoring this gives ax = (x-y)(x-z)

In simlar manner, we obtain

by = (y-z)(y-x) and cz = (z-x)(z-y)

Now, let (x,y,z) be a solution. Without loss of peneality we may assume $x \ge y \ge z$. Then by = $\{y-z\} \{y-x\} \le 0$ and $cz = \{z-x\} \{z-y\} \ge 0$ and the conditions b > ax > 0 imply y $\le 0 \le z \le y$. Thus, y = z = 0 and $ax = x^2$. So the solutions is this case are (0,0,0), (0, b, 0) and (0, 0, c)

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TYPES OF TRIANGLES

A.SAIRAM, 12[™] Class, Khammam, T.S

The **different types of triangles** are classified according to the length of their sides and as per the measure of the angles. The triangle is one of the most common shapes and is used in construction for its rigidity and stable shape. Understanding these properties allows us to apply the ideas in many real-world problems.

What are the Different Types of Triangles?

There are different types of triangles in math that can be distinguished based on their sides and angles.

Types of Triangles Based on Sides

On the basis of side lengths, the triangles are classified into the following types:

Equilateral Triangle: A triangle is considered to be an equilateral triangle when all three sides have the same length.



Isosceles triangle: When two sides of a triangle are equal or congruent, then it is called an isosceles triangle.



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Scalene triangle: When none of the sides of a triangle are equal, it is called a scalene triangle.



Types of Triangles Based on Angles

On the basis of angles, triangles are classified into the following types:

Acute Triangle: When all the angles of a triangle are acute, that is, they measure less than 90°, it is called an acute-angled triangle or acute triangle.

Right Triangle: When one of the angles of a triangle is 90°, it is called a right-angled triangle or right triangle.



Obtuse Triangle: When one of the angles of a triangle is an obtuse angle, that is, it measures greater than 90°, it is called an obtuse-angled triangle or obtuse triangle.

Types of Triangles Based on Sides and Angles The different types of triangles are also classified according to their sides and angles as follows:

Equilateral or Equiangular Triangle: When all sides and angles of a triangle are equal, it is called an equilateral or equiangular triangle. In it each angle is 60°.

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Isosceles Right Triangle: A triangle in which 2 sides are equal and one angle is 90° is called an isosceles right triangle. So, in an isosceles right triangle, two sides and two acute angles are congruent. The angles are 45°.45°.90°.

Obtuse Isosceles Triangle: A triangle in which 2 sides are equal and one angle is an obtuse angle is called an obtuse isosceles triangle.

Acute Isosceles Triangle: A triangle in which all 3 angles are acute angles and 2 sides measure the same is called an acute isosceles triangle. Angles oposite to equal sides are equal.

Right Scalene Triangle: A triangle in which any one of the angles is a right angle and all the 3 sides are unequal, is called a right scalene triangle.

Obtuse Scalene Triangle: A triangle with an obtuse angle with sides of different measures is called an obtuse scalene triangle. The other two angles are Acute angles.

Acute Scalene Triangle: A triangle that has 3 unequal sides and 3 unequal acute angles is called an acute scalene triangle.

Important Notes:

Here is a list of a few points that should be remembered while studying the types of triangles:

- In an equilateral triangle, each of the three internal angles is 60°.
- The three internal angles in a triangle always add up to 180°.
- All triangles have at least two acute angles.

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THE MISSING NUMBERS



Dr.K. Pushpalatha, Associate Professor, KLUniversity

What are the Missing Numbers?

Missing numbers are the numbers that have been missed in the given series of a number with similar differences among them. The method of writing the missing numbers is stated as finding similar changes between those numbers and filling the missing terms in the specific series and places. In this article we will learn what are missing numbers, missing number series, how to find missing numbers in the series and sequence with examples, solved examples on missing numbers in the series and sequence, etc.

Missing Number Series

We have seen that number series is a collection of numbers that follows a particular rule or formula. There are various types of series and missing number series is one among them. In missing number series, a series is given with one missing number and you are asked to find the missing term. To find the missing number, we first identify the rule or formula which is applied in the given missing number series. Let us learn the method to find the missing number in a series.

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How to Find The Missing Number in a Series?

In the given missing number series, you can sometimes find missing numbers at the beginning or at the end of the series. The layout in the missing number series is similar to the wrong number series, you have to identify the rule and then use the rule to estimate the next number.

For better understanding we will classify Number Series into the following broad categories

- Difference series
- Product series
- Squares/Cubes series
- Miscellaneous series
- Combination series

Difference Series:

The difference series can be further classified into

Number series with constant difference

In the number series with constant difference, there is always a constant difference between two consecutive numbers. For example, the numbers of the series 1, 4, 7, 10, 13 are such that any number is obtained by adding a constant figure 3 to the preceding term of the series.

Number series with increasing or decreasing difference

In the series with increasing/decreasing difference, the difference between consecutive terms keeps increasing(or decreasing, as the case may be).For example, let us try to find out the next number in the series 2,3,5, 8,12,17,23,.....

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Product Series:

A product series is usually a number series where the terms are obtained by a process of multiplication .Here also there can be different types of series. We will look at these through examples.

Consider the series 2, 4, 8, 16, 32, 64...... Here, each number in the series is multiplied by 2 to get the next term. So, the term that comes after 64 is 128.So, each term is multiplied by a fixed number to get the next term.

Squares/Cubes Series:

There can be series where all the terms are related to the squares of numbers or cubes of numbers. With squares/ cubes of number as the basis, there can be many variations in the pattern of the series. Let us look at various possibilities of series based on squares/cubes.

Each term of the series may be the square of a natural number, such as 1, 4, 9, 16 ... The numbers are squares of 1, 2, 3, 4 respectively. The number which follows 16(which is the square of 4) will be 25(which is the square of 5).

Miscellaneous Series:

There are series that do not come under the other patterns and are of general nature but are important and are fairly common. Even here, sometimes, there can be a specific pattern in some cases.

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Take the series 3, 5, 7, 11, 13 ...this is the series of PRIMES NUMBERS. It is an important series and the student should look out for this as one of the patterns. The next term in this series is 17.

There can also be variations using prime numbers. Take the series 9, 25, 49, 121In this series, the terms are SQUARES of prime numbers . Hence, the next term is 13², i.e., 169.

Combination Series:

A number series which has more than one type of (arithmetic) operation performed or more than one series combined together is a combination series .The series that are combined can be two series of the same type or could be different types of series described above .Let us look at some examples.

First let us look at series that are formed by more than one arithmetic operation performed on the terms to give the subsequent terms.

Consider the series 2, 6, 10, 3, 9, 13, 4, 12,Here, the first term 2 is multiplied by 3 to get the second term, and 4 is added to get the third term. The next term is 3(one more than the first term 2) and it is multiplied by 3 to get 9 (which is the next term)and then 4 is added to get the next term 13.The next term 4(which is one more than 3)which is multiplied with 3 to get 12, the next term. Then 4 is added to this to get the next number 16.

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Here Are The Steps To Find The Missing Numbers in a Series

- 1. Select 2 or 3 terms in the text which rule will be applied to find the missing number. For example: If you have 5 numbers in a series then pick the first 3 terms to check the rule that is to be applied.
- 2. While choosing the number to check the rule, select the number that is easy to operate. These include terms that are factors of 2, 3, 5, or 10. Check the series with some common methods such as the sum of the terms, squares, cube, or other.

Let Us Understand Through An Example: Find the missing number 1, 2, 6, 24,?

Solution: The given sequence has 4 terms. We will check which rule is applied by picking the first 3 terms. The second number in the sequence is 2 and the first number is 1 which means 1 is either added or 2 has been multiplied to obtain the second term. The third term is 6 which we got from 2 by multiplying with 3. Hence, now we have 1×1 , 1×2 , 2×3 , and 6×4 . Thus, we have identified the rule and accordingly, the last term will be $24 \times 5 = 120$.

Hence, the missing number is 120

How To Find A Missing Number In a Sequence?

Here are the steps to find missing numbers in sequence:

 Identity, if the order of number given is ascending (smaller to larger number) or descending (larger to smaller number)

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- 2. Calculate the differences between those that are next to each other.
- 3. Estimate the difference between numbers to calculate the missing number.

Let us Understand The Above Steps Through An Example:

Find The Missing Number in the Following Sequence 30, 23, ?, 9.

Solution: The numbers given in sequence are in decreasing order. It implies that numbers are arranged from larger to smaller.

The difference between the numbers 30-23=7

As the sequence of the numbers is in decreasing order, subtract 7 from 23. The missing number is 16 as it is 7 more than the previous number 9. So the missing number is 16.

Solved Examples

Find the missing number in the following sequence 3, 5, 7, 11, ? 17, 19

Solution:

The missing number found in the following sequence is 13.

It is because all the given numbers in the sequence 3, 5, 7, 11, 17, 19 are prime numbers. The numbers given in the sequence are prime numbers as they can be divided only by 1 and itself.

Hence, the number line series will be 3, 5, 7, 11, **13**, 17, and 19.

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2. Find the missing number in the following sequence 1, 3, 9, 15, 25

Solution:

The missing number found in the following sequence is 35.

It is because all the numbers in the sequence are squares and (square-1) such as

 $1^2 = 1$; $2^2 = 4$ and then 4 - 1 = 3; $3^2 = 9$; $4^2 = 16$ and then 16 - 1 = 15; $5^2 = 25$ and $6^2 = 36$ and then 36 - 1 = 35; $7^2 = 49$

Hence, the number line series will be 1, 3, 9, 15, 25, 35, and 49

PROBLEMS:

Find the next number in the series



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ఒక చక్కటి గణిత శ్లోకం

B.B. Rama Sarma

యధాశిఖా మయూరాణాం

నాగానాం మణయోయథా

తద్వద్వేదాంగ శాస్తాణాం

గణితం మూర్థిని స్థితం

(వేదాంగ జ్యోతిషం గ్రంథం నుండి)

గణిత టియులారా! పైన శ్లోకం గణితశాస్త్రప్రాముఖ్యతను చక్కగా తెలియజేస్తున్నది.

"నెమళ్ళకు పింఛమువలె, సర్పాలకు మణివలె వేదాంగశాస్త్రాలన్నిటికినీ గణితశాస్త్రం శిరస్థానంలో భాసిస్తున్నది" అని ఈ శ్లోకం అర్థం.

కాబట్టి గణిత శాస్త్రపు ఆలంబన లేకుండా ఇతర విజ్ఞాన శాస్త్రాలను ఆకళింపు చేసికొనజాలము.

అందులకే ఇతర శాస్ర్రాలన్నీ పూలనుకుంటే, గణితం అంతర్లీనమైన దారమవుతుంది. దారం లేకుండా దండనిలువదు కదా. దారం బయటకు కనబడనంత మాత్రాన అదిలేదనుకోవడం అజ్ఞానమేకదా! కాబట్టి గణిత ప్రాముఖ్యతను అందరూ గ్రహింతురుగాక! అందరికీ చాటుదురు గాక!

ధన్యవాదములు

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Sieve of Eratosthenes - Prime Numbers 1 to 100

– B. Ritwik, 12th Class, Khamman

Now here we are going to encircle the prime numbers from 1 to 100 using the Sieve of Eratosthenes method. As we have already discussed the prime and composite numbers it will be more convenient to find the difference between them.

The steps involved in separating the prime numbers from 1 to 100 are as follows.

* Step 1:	First, write all the natural numbers from 1 to
	100, row-wise and columnwise, as shown in
	the below figure.
* Step 2:	Put a cross over 1, as it is neither a prime
	nor a composite.
* Step 3 :	Now, encircle the number 2 (which is a prime
	number) and cross all the multiples of 2, such
	as 4, 6, 8, 10, 12 and so on. Since all the
	multiples of 2 are composite.
* Step 4 :	Next, encircle the number 3, and put a cross
	over all the multiples of 3, such as 6,9, 15,
	21 etc. Since apart from 3, all its multiples
	are composite.
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* Step 5 :	Again, encircle the number 5 (since it has only two factors), and put a cross over all
	the multiples of 5.
* Step 6 :	Now encircle 7 and cross all the multiples of
	7.

- *** Step 7**: Encircle 11 and cross all the multiples of 11.
- * Step 8: Continue the proces unless all the numbers are either encircled or crossed.



Therefore, we have concluded that all the encircled numbers are prime numbers and all the crossed numbers are composite numbers. So, the prime numbers from 1 to 100 are:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, and 97

Hence, the Sieve of Eratosthenes method is completed.

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OLYMPIAD CORNER - V. Pavan Kumar M.Tech Hyderabad 1. Suppose a, b denote the distinct real roots of the Quadratic Polynomial x² + 20x – 2020 and suppose c,d denote the distinct complex roots of $x^2 - 20x +$ 2020. Find the value of ac (a - c) + ad (a - d) + ad (abc (b - c) + bd (b - d).**Sol.** $x^2 + 20x - 2020 = 0$ has roots a,b ∴ a + b = - 20, ab = - 2020 $x^2 - 20x + 2020 = 0$ has roots c,d \therefore c + d = 20, cd = 2020 **Required expression** = ac(a-c) + ad(a-d) + bc(b-c) + bd(b-d) $= a^{2}c - ac^{2} + a^{2}d - ad^{2} + b^{2}c - bc^{2} + b^{2}d - bd^{2}$ $= (a^2+b^2) [c+d] - (a+b) (c^2+d^2)$ $= 20 (a^2 + b^2 + c^2 + d^2)$ $= 20 [(a+b)^2 + (c+d)^2 - 2ab - 2cd)]$ = 20 (400 + 400 + 4040 - 4040) = 20 × 800 = 16000

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2. Let the function
$$f : [0,1] \to R$$
 be defined by $f(x) = \frac{4^{x}}{4^{x}+2}$. Find the value of $\sum_{r=1}^{39} f\left(\frac{r}{40}\right) - f\left(\frac{1}{2}\right)$
Sol. $f(x) + f(1-x)$
 $\frac{4^{x}}{4^{x}+2} + \frac{4^{1-x}}{4^{1-\lambda}+2} = \frac{4^{x}}{4^{x}+2} + \frac{4^{1-x}4^{x}}{4+2.4^{x}}$
 $= \frac{4^{x}}{4^{x}+2} + \frac{4}{2(4^{x}+2)} = \frac{4^{x}+2}{4^{x}+2} = 1$
 $\therefore f(x) + f(1-x) = 1$
Now $f\left(\frac{1}{40}\right) + f\left(\frac{39}{40}\right) = 1$
 $f\left(\frac{2}{40}\right) + f\left(\frac{38}{40}\right) = 1$
 $f\left(\frac{19}{40}\right) + f\left(\frac{21}{40}\right) = 1$ and $f\left(\frac{20}{40}\right) + f\left(\frac{20}{40}\right) = 1$



4. The sides of a right angled triangle are in A.P. It's area is 24. Find the length of the smallest side. Sol. Let the sides be a - d, a, a+d; d > 0 (a + d)² = (a - d)² + a² a² + 2ad + d² = a² - 2ad + d² + a² a² + 2ad + d² = a² - 2ad + d² + a² 4ad = a² ⇒ a = 4d Also 1/2 · a · (a - d) = 24 ⇒ a (a - d) = 48 4d × 3d = 48 ⇒ d = 2; a = 4d = 8 Sides are 6, 8, 10 Smallest side = 6 Let L₁, L₂ L₁₀₀ be consecutive terms of an AP with common difference d₁. Let W₁, W₂ W₁₀₀ be

common difference d_1 . Let W_1 , W_2 W_{100} be consecutive terms of another A.P with common difference d_2 . It is given that $d_1d_2 = 10$. Now for each i = 1, 2 100, Let R_L be a rectangle with length L_1 , width W_1 and area A_1 . If A_{51} - $A_{50} = 1000$, Find the value of A_{100} - A_{90} . Sol. $A_{51} - A_{50} = 1000 \Rightarrow L_{51}W_{51} - L_{50}W_{50} = 1000$ $\Rightarrow (L_1 + 50d_1) (W_1 + 50d_2) - (L_1 + 49d_1) (W_1 + 49d_2) = 1000$

$$L_1 d_2 + W_1 d_1 + 99 d_1 d_2 = 1000$$

 \Rightarrow L₁d₂+W₁d₁= 10

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Now
$$A_{100} - A_{90} = (L_1 + 99d_1) (W_1 + 99d_2) - (L_1 + 89d_1)$$

 $W_1 + 89d_2) = 10 (d_1W_1 + L_1d_2) + 1880d_1d_2$
 $= 100 + 18800 = 18900$
6. Find the product of all real postive values of x satisfying the equation.
 $x^{(16(\log_5 x)^3 - 68\log_5 x)} = 5^{-16}$
Sol. Taking log to the base 5 both sides, we get
 $(16t^3 - 68t) t = -16 \log_5^5$ where $\log_5 x = t$
 $16t^4 - 68t^2 = -16$
 $4t^4 - 17t^2 + 4 = 0$
 $4t^4 - 16t^2 - t^2 + 4 = 0$
 $(4t^2 - 1) (t^2 - 4) = 0$
 $t = \pm \frac{1}{2}, \pm 2$ $\therefore \log_5 x = \pm \frac{1}{2}, \pm 2$
 $x = 5^{1/2}, 5^{-1/2}, 5^2, 5^2$
 $x_1 \cdot x_2 \cdot x_3 \cdot x_4 = 5^{1/2} \times 5^{-1/2} \times 5^2 \times 5^{-2}$
 $= 5^{\frac{1}{2} - \frac{1}{2} + 2^{-2}} = 5^\circ = 1$
 $\pm * \pm *$













CLASS - VIII									
1.	Which of the following set of numbers is closed								
	under division.								
	1) N	2) w	3) R	4) none					
2.	Which of the following is negative of -2.								
	1) –2	2) ± 2	3) ¹ / ₂	4) 2					
3.	The reciprocal of a +ve number is								
	1) negative	2) positive	3) ±	4) none					
4.	The rational	number that	t is equal to it	s negative.					
	1) 1	2) 0	3) 2 <i>/</i> 0	4) none					
5.	Which of the	e following n	umbers are ec	ual to their					
	own reciprocals.								
	1) 1	2) -1	3) 0 4) 1 an	d 2 are true					
6.	Which of th	e following r	ational numb	er does not					
	lie between 3/5 and 3/4.								
	1) 97 160	2) 98 160	3) 99 160	4) 96 120					
7.	Which of the following is a linear expression.								
	1) 2x	2) x ²	3) x + x ²	4) x ² – x					
8.	The differen	ce between t	wo numbers is	66 and their					
	ratio is 2 : 5. Then the two numbers are								
	1) 44,110		2) 11,77						
	3) 22,88	_	4) 33,99						
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9. A grand father is ten times older than his grand daughter. He is also 54 years older than her. Their present ages are _____ 3) 10,64 1) 6,60 2) 8,62 4) none 10. If 0.25(4x-3)=0.05(10x-9) then x = ____ 4) 0.6 1) 2.4 2) 2.0 3) 2.6 11. Present ages of Anu and Raj are in the ratio 4 : 5. Eight years from now the ratio of their ages will be 5:6. Their present ages are 1) 32,40 2) 28,35 3) 20,25 4) 24,30 12. $\frac{x+1}{2x+3} = \frac{3}{8}$ then 1) x = 2 2) $x = \frac{1}{2}$ 3) x = -2 4) $x = -\frac{1}{2}$ 13. If $\sqrt{x-3} = y$, $\sqrt{y-4} = z$ and $\sqrt{z-5} = 2$ then x =_____ 1) 6129 2) 6565 3) 7103 4) 7228 14. The sum of the measures of the external angles of any polygon is ____ degrees. 4) 90 1) 180 2) 360 3) 270 15. The measure of each exterior angle of a regular polygon of 15 sides is __degrees. 1) 25 2) 24 3) 27 4) 30 40 Ganitha Chandrika (ISSN 0973-3493) Vol.23 (3&4) July-December 2022

CLASS - IX								
1.	Which is t	he missing	term in the	sequence				
	1,3,6,10,15,_							
	1)18	2)21	3)20	4)32				
2.	Average of f	irst 100 natuı	ral numbers is <u></u>					
	1)50.5	2)100.5	3)28.5	4)58.5				
3.	Degree of (1	+ x)(1 + x ²)(1	+ x ⁴)(1 + x ⁸)	s				
	1)8	2)15	3)9	4)64				
4.	Number of	quadratic fa	ctors for (x ⁴ +	• x ² + 1) ² is				
	1)4	2)8	3)2	4)0				
5.	$\left(2+\sqrt{3}\right)^2\left(2-\sqrt{3}\right)^2$	$\sqrt{3}$) ² $(1+\sqrt{2})^{2}$	$(1-\sqrt{2})^2 = $					
	1)18	2)64	3)1	4)0				
6.	$3x^2 + 4y^2 = 10$	$0, 4x^2 + 3y^2 = 3$	11 <i>then x</i> ² + y ² :	=				
	1)121	2)21	3)3	4)7				
7.	$x + x^{-1} = 2.5 t$	then x ³ + x ³ =						
	1)8.125	2)6.125	3)1	4)7.25				
8.	Distance bet	ween (3, -4) a	nd (6, 0) is	units				
	1)2	2)7	3)5	4)8				
9.	$\frac{\mathbf{a}+\mathbf{b}}{\mathbf{c}+\mathbf{d}}=\frac{\mathbf{a}-\mathbf{b}}{\mathbf{c}-\mathbf{d}}\Longrightarrow$	ad – bc + 1 =	=					
	1) 0	2)1	3)2	4)-2				
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10.
$$x^4 - x^2 + 1 = 0 \Rightarrow (x^2 + x^{-2})^8 =$$

1)1 2)0 3)16 4)256
11. $2(x-1)(x-7) = ax^2 + bx + c \Rightarrow 4a + 2b + c =$ _________
1)40 2)-10 3)0 4)-12
12. $\sqrt{x} + \sqrt{y} = \sqrt{4 + 2\sqrt{3}} \Rightarrow x + y =$ ________ number.
1)even 2)odd
3)odd prime 2)odd
3)odd prime 4)even prime
13. Area of an equilateral triangle whose height is $\sqrt{12}$
is ______Sq.units
1) $3\sqrt{3}$ 2) $2\sqrt{3}$
3) 3 4) $4\sqrt{3}$
14. In a right-angled triangle, the legs are 3, 4. Then its perimeter is _______
1)12 2)10 3)14 4)7
15. Area of a semi-circle is 18π Sq.u. Its circumference is _______
1) $\pi + 2$ 2) $6\pi + 12$
3) $9\pi + 3$ 4) 3π

CLASS - X 1. Number of integer pairs (x, y) such that $x^2 + y^2 - 2x$ -4y + 5 = 0 is n then $n^2 - 5n + 6 = ____$ 1) 2 2) 0 3) 4 4) 7 2. $2 - \sqrt{3}$ is a root of $x^2 + ax + b = 0$ then (a+1) (b+1) = _ Where a, b∈Z 1)-3 3)0 2)-6 4)8 3. a+b+c=0 then $a^6+b^6+c^6+2a^3b^3c^3\left[\frac{1}{a^3}+\frac{1}{b^3}+\frac{1}{c^3}\right]$ $-9a^{2}b^{2}c^{2}+1=.....$ 1)8 2)0 3)1 4)24 4. S_{11} , S_{22} are inscribed and Circumscribed circles of a square of side unity. Then difference of areas of these circles is 2) $\frac{\pi}{3}$ 1) $\frac{\pi}{4}$ 3) $\frac{\pi}{6}$ 4) π 5. Mean of first 2020 odd natural numbers is 1)1010 2)2021 3)4040 4)None of these 6. $1^2 \cdot 2^2 + 3^2 \cdot 4^2 + \dots 1997^2 - 1998^2 = A$ then $\frac{A}{1999} =$ 1)-1000 2)-999 3)1001 4)999 7. Base of a regular hexagon has ends (0, 0), (3, 4). Its area is $x\sqrt{3}$ then $\frac{2x}{25}$ =____ 1)2 2)3 3)4 4)5 Ganitha Chandrika (ISSN 0973-3493) Vol.23 (3&4) July-December 2022





National Mathematics Day Celebrations

National Mathematics Day is celebrated on 22nd December every year to mark the birth aniversary of legendary Indian Mathematician, **Sri Srinivasa Ramanujan**. The Indian Government declared 22nd December to be National Mathematics Day. It was introduced by the then Prime Minister Manmohan Singh on 26th December 2011 at Madras University to mark the 125th birth anniversary of the Indian Mathematician Sri Srinivasa Ramanujan.

* KBN College, Vijayawada. Department of Mathematics & Statistics organized two day celebrations named Mathophilia - 2022 which were held on 21st and 22nd December 2022.

The main objective of the programme is to create awareness and enhance Mathematical skills among studens and to erase the phobia towards the subject by creating a practical knowledge through different aspects. The main motto of this programme is to make students aware of the fact that Mathematics can be used in different forms such as Models Expo. Arts, Games & Cultural Activities etc.

On 21st inaugural session was conducted in which Shri S.DilliRao IAS, Collector, NTR District inaugurated **Math Expo - 2022** Math Expo was arranged by exhibiting the

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Mathematical Plyaing cards and around 65 Mathematical models preparedby our Degree students. 4000 students and 50 teachers from different schools of Vijayawada visited the Expo. In the afternoon Ganitha Ashtavadhanam was conducted by Dr. T.S.V.S. Suryanarayanamurthy, Ganitha Avadhani, Amalapuram. On 22nd Competitions such as Problem Solving Session, Poster Presentation, Power point presentation and Elocution were conducted. 170 students from 15 Degree and Engineering Colleges in Krishna & Guntur districts participated in the competetions. In the afternoon Valedictory Sesson was started with garlanding the portrait of Srinivasa Ramanujan. Maths Cultural Events such as Ganitha Pelli Patrika, Math Songs, GanithaSankranthi and 4x4 Magic Square Dance were performed by Degree students.

In this session Srinivasa Ramanujan Memorial Award was presented to Prof. D.S.N. Sastry. Later prizes to the winners and participation certificates were distributed. Our beloved Principal Dr. V. Narayana Rao, Secretary and Correspondent T.Srinivas. President T. Sheshaiah, Chief Guest of the session Sri T.V. RameshBabu MD, C- Channel, Concept Head Dr. P. Satyanarayana Sarma, Head Dept. of Mathematics Smt. M. Lakshmi Prasanna and the faculty members of Mathematics and Statistics participated in this session. The session ended with the formal vote of thanks by P. Kalma Begum, lecturer in Mathematics.

- Department of Mathematics and Statistics, K.B.N. College

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* పిడుగురాళ్ళలోని వివేకానంద పాఠశాలలో గణితదినోత్సవాన్ని పురస్కరించుకొని A.I.M.Ed, విజయవాడవారిచే నిర్వహించబడిన MSET - 2022 పరీక్షలో రాంకులు సాధించిన విద్యార్థులకు బహుమతి ప్రదానోత్సవం నిర్వహించారు. ఈ కార్యక్రమానికి ముఖ్య అతిథులుగా A.I.M.Ed LIfe Time President Pro. భవనారి సత్యనారాయణగారు, రిటైర్డు గవర్నమెంటు కాలేజీ ప్రిన్సిపాల్ జ్రీ ముత్తారెడ్డిగారు, గౌరవ అతిథులుగా విట్ కాలేజి ప్రిన్సిపాల్ జ్రీరాఘవగారు విచ్చేసారు.

భవనారి సత్యనారాయణగారు మాట్లాడుతూ వివేకానందపాఠశాల పిల్లలు చక్కని [పతిభ కనపరిచారని, వారిని, వారికి శిక్షణ ఇచ్చిన మాష్టారు శ్రీరామకృష్ణగారిని అభినందించారు. ఈ విజయం సంస్థకి, తల్లిదండ్రులకి, పట్టణానికి గూడ గర్వకారణమని కొనియాడారు. జిల్లా మరియు మండలస్థాయిలో రాంకులు కైవసం చేసుకున్న 15మంది విద్యార్థులను మెమెంటోలు మరియు సర్హిఫికెట్లతో సత్తరించారు.

* విజయవాడ, సత్యనారాయణపురంలోని శ్రీ విజ్ఞానవిహార్ (E.M) హైస్కూల్లో పాఠశాల (పిస్పిపాల్ శ్రీమతి J.V. షనత్ కృష్ణగారి అధ్యక్షతన సమావేశం నిర్వహించారు. ముఖ్య అతిధులు శ్రీ R. శ్రీధర్, Rtd. H.M (ఎడిటర్, గణితచంద్రక), శ్రీ P.S.N. మూర్తి, శ్రీచక్రధర్లు విచ్చేశారు.

అతిధులచే జ్యోతి ప్రజ్వలన అనంతరం IX, X విద్యార్థులకు క్విజ్, Logical ప్రశ్నల పోటీని గణితోపాధ్యాయని శ్రీమతి లలిత నిర్వహించారు. విద్యార్థులు కొన్ని గణిత సంబంధ పాటలను పాడారు. విజేతలను పాటలు పాడిన వారిని అభినందించి Sweets పంచారు.

అతిథులు తమ సందేశంలో గణితం నిత్యజీవితంలో ఒక భాగమని, గణితం లేనిదే మనుగడ లేదని వివరించారు. కొంతమంది గణితం అంటే భయపడతారని, కాని దానిని మనస్సుపెట్టి (శద్ధగా (పయత్నిస్తే మంచి మార్కులు సాధిస్తారని వివరించారు. గణితాన్ని ఒక పాఠ్యాంశంగానే కాక వినోదభరితంగా కూడ చూడవచ్చని, సూడో, పజిల్సు, మాజిక్ చదరాలు మొదలైనవి అటువంటివేనన్నారు. పోటీ పరీక్షలలో పాల్గొంటే మెదడు మంచి పదును పడుతుందన్నారు. శ్రీనివాసరామానుజన్ సాధించిన కొన్ని అంశాలను వివరించారు. టిన్సిపాల్గారు అతిథులని సత్మరించారు. ఈకార్యక్రమాన్ని శ్రీమతి లలిత, శ్రీమతి అనూరాధలు నిర్వహించారు. - లలిత, గణితోపాధ్యాయిని

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